The Next Dimension: Web-Based Visualization of 4D Cube Unfolding

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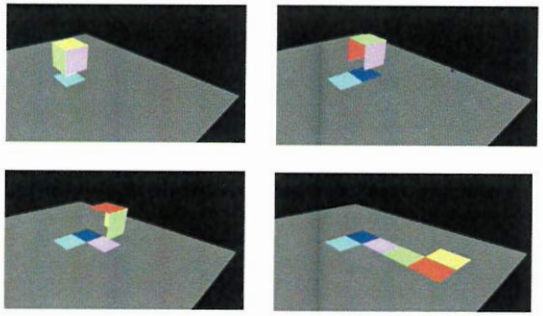
Abstract

The purpose of this research project is to use the mathematical power of abstraction to create web visualizations of 3-dimensional cube and 4-dimensional hypercube unfolding. The term 'unfolding' means representing an n-dimensional object in (n-1)-dimensional space. The ability to create visuals of a 4-dimensional object is important because such an object cannot be seen in the real, 3-dimensional world. While a 3D cube unfolds into six squares a 4D hypercube (tesseract) unfolds into eight cubes. In order to solve the problem of hypercube unfolding we first developed a program that visualized the unfolding of a 3-dimensional cube. We designed an unfolding algorithm for the 3D cube, which allowed only the legal moves and produced all possible shapes of the resulting 2D object. We then extended the 3D algorithm to the **next dimension**, 4D, and obtained a program that allows the user to produce and see all legal tesseract unfoldings. The algorithm uses an innovative approach based on the concept of an associated (hyper)cube graph. To develop the fully interactive Web-based application that can be accessed from anywhere in the world we used Javascript, HTML and CSS.

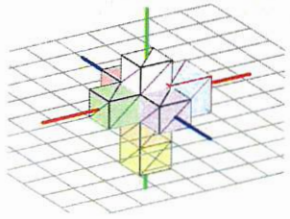
Unfolding



If you have a square, (a two-dimensional object), unfolding it would mean selecting one side to stamp on the ground, then rotating the square to stamp the next side on the ground, and this repeats until all four sides have made contact with the ground. The square completely unfolded is a line, (a one-dimensional object). If you have a three-dimensional cube, when you unfold it, you stamp it side by side. When it is completely unfolded, it is a formation of squares connected to each other by at least one side. The unfolded object is always one dimension lower than the original object. From this pattern, we know that a four-dimensional cube will unfold into a formation of three-dimensional cubes. The process of creating such a formation is shown on the sequence of eight slides shown to the right.

Complete 4D Unfolding

Graph (Hypercube Dual)

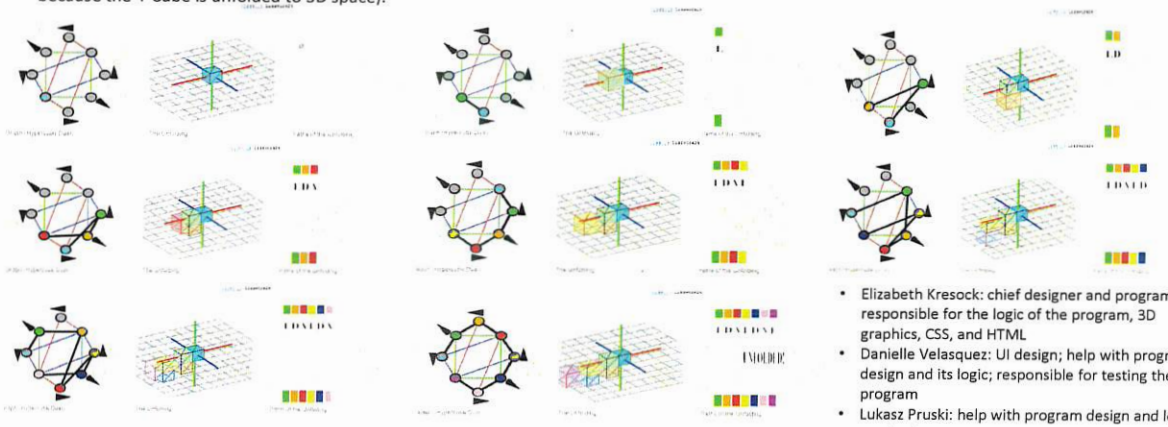


The Unfolding

Abstraction at Work

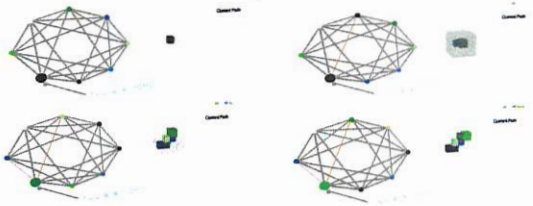
In our three-dimensional world a four-dimensional cube does not exist. Mathematical abstraction gives us the power to visualize objects that "do not exist." We have developed a program that illustrates how a 4-Cube unfolds into a structure of 3-Cubes. We first developed a program that unfolds a 3-Cube into a formation of squares (2-Cubes) in the plane. The screenshots on the left show a few stages of the process of unfolding. Note that the unfolding proceeds in steps that can be taken in four directions: right, left, up, and down (four directions in two dimensions because the 3-Cube is unfolded to a plane):

Then, using the power of abstraction, we extended the algorithm from three dimensions to four. The following screenshots illustrate the process of unfolding. Note that the unfolding proceeds in steps that can be taken in six directions: right, left, up, down, toward, and away (six possible directions because the 4-Cube is unfolded to 3D space):



3D Graphics -Three.js

The web-based, interactive program was written in JavaScript, with CSS and HTML components of the code. To provide three-dimensional graphics we used Three.js. It is an open-source JavaScript library and Application Programming Interface (API) that creates and displays 3D graphics in a web browser. The programmer can create cameras, lights, geometries, objects, materials, shaders, scenes, etc. and then choose whether to render the scene with HTML 5 canvas, WebGL or SVG.



Program Functionalities

The program allows the user to create various sequences of unfolding steps by selecting the directions. These features include:

- Select the direction of the next unfolding move
- Undo the previous step
- Illustrate the unfolding process on the associated graph (dual) of the 4-Cube
- Display the "unfolding path" (sequence of steps taken by the user)
- Demonstrate the entire process of unfolding on a pre-coded sequence of steps ("Play the unfolding movie")

- Elizabeth Kresock: chief designer and programmer; responsible for the logic of the program, 3D graphics, CSS, and HTML
- Danielle Velasquez: UI design; help with program design and its logic; responsible for testing the program
- Lukasz Pruski: help with program design and logic; mentoring.

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