## DIFFRACTION RIDGE PROBLEMS (SINGLE EDGED DIFFRACTION)

Consider a uniform plane wave incident on a perfectly conducting half-plane as in Figure 1. We want to calculate the electric field at a point *P* a distance *r* behind the plane.

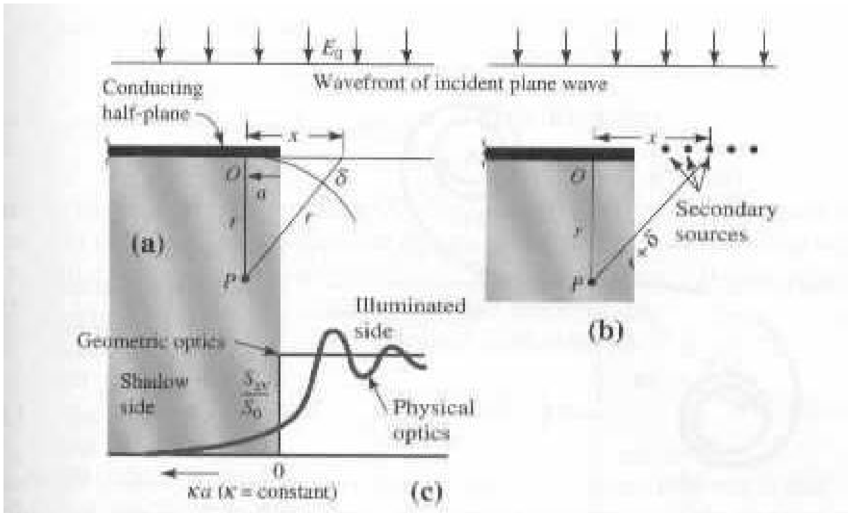


Figure 1. Plane Wave Incident on Conducting Plane - Cornu Spiral

The resulting electric field have the form of Fresnel integrals and can be written

$$E = \frac{E}{kr} e^{-jbr} \left\{ \frac{1}{2} + \frac{1}{2} j - \left[ C(ka) + jS(ka) \right] \right\} \qquad (4\text{-}16\text{-}7)$$

where $$C(ka) = \int_0^{ka} cos \frac{pu^2}{2} du \text{ = Fresnel cosine integral} \qquad (4\text{-}16\text{-}8)$$

$$S(ka) = \int_0^{ka} sin \frac{pu^2}{2} du \text{ = Fresnel sine integral} \qquad (4\text{-}16\text{-}9)$$

and $$ka = \sqrt{\frac{2}{rl}} a \text{ , dimensionless.}$$

A graph of $C(ka)$ and $S(ka)$ yields the Cornu Spiral. Since $C(-ka) = -C(ka)$ and $S(-ka) = -S(ka)$, the spiral for the negative values of $ka$ is in the third quadrant and is symmetrical with respect to the origin for the spiral in the first quadrant.

The Power density as a function of $ka$ is then

$$S_v = \frac{EE^*}{Z} = S_o \frac{1}{2}\left\{\left[\frac{1}{2} - C(ka)\right]^2 + \left[\frac{1}{2} - S(ka)\right]^2\right\} \text{ [W/m}^{-2}\text{] ,} \qquad (4\text{-}16\text{-}10)$$

where $$S_o = \frac{E_o^2 l}{2Zr} \text{ [W/m}^{-2}\text{] .} \qquad (4\text{-}16\text{-}11)$$

The power density variation as a function of $ka$ is shown in Figure 1. Assuming that the plane wave originates from a distant source,

1. For no obscuration, $ka = -\infty$ and $S_{av} = S_o$ .
2. For source, observer and edge of obscuring plane in line, $ka = 0$ and $S_{av} = ¼ S_o$ .
3. For complete obscuration, $ka = +\infty$ and $S_{av} = 0$ .

The power density does not go to zero abruptly as the point of observation goes from the illuminated side to the shadow side.

The relative power density as a function of $ka$ is

$$S_{av}\,(relative) = \frac{S_{av}}{S_o} = \frac{1}{2}\left\{\left[\frac{1}{2} - C(ka)\right]^2 + \left[\frac{1}{2} - S(ka)\right]^2\right\} \qquad (4\text{-}16\text{-}12)$$

The relative power density is equal to ½ $R^2$, where $R$ is the distance from a $ka$ value to the Cornu Spiral to the point (½,½). For large positive values of $ka$, $R$ approaches $\frac{1}{p\,ka}$, so that

$$S_{av}\,(relative) \approx \frac{1}{2}\left(\frac{1}{p\,ka}\right)^2 = \frac{r\,l}{4p^2 a^2} \text{ ,} \qquad (4\text{-}16\text{-}13)$$

where $r$ = distance from obstacle (conducting half-plane) [m]
$\lambda$ = wavelength [m]
$a$ = distance into the shadow region [m].

Equation(4-16-13) can be used when $ka > 3$ .

Example:

A vertical conducting wall 25 m high extends above a flat ground plane. A 10 cm wavelength transmitter is situated 25 m above ground at a large distance (far-field) to one side of the vertical wall and the receiver is located on the ground 100 m to the other side of the wall. Find the signal level at the receiver due to diffraction over the wall as compared to the direct path signal without the wall.

*Solution*: The constant $k = \sqrt{\frac{2}{r\,l}} = \sqrt{\frac{2}{100(0.1)}} = 0.44$ and $a$ = 25 m, so $ka$ =11 which is greater than 3. So,

$$S_{av}\,(relative) \approx \frac{1}{2}\left(\frac{1}{p\,ka}\right)^2 = \frac{r\,l}{4p^2a^2} = \frac{1}{2500}$$ or -34 dB.

Thus the vertical wall causes 34 dB of attenuation as compared to a direct path signal