

Multiport Parameter Matrices

For a one-port device, we know that we can express the relationship between V and I in either impedance ($Z = R + jX$) or admittance ($Y = G + jB$) forms. We make the choice based on the nature of the network that comprises the one-port, typically choosing Z for series elements and Y for parallel elements.

For a two-port we have a wider range of choices, always restricted to two equations defining the four variables, V_1 , I_1 , V_2 and I_2 .

We can express the relationships in z, y, h, g or ABCD parameters:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$V_1 = A V_2 + B I_2$$

$$I_1 = C V_2 + D I_2$$

We can determine the values of the various parameters by measurements that are made under the conditions that constrain other parameters to be zero. For example, using z parameters, we can set $I_2 = 0$ by open circuiting the output of the two-port. This allows us to measure z_{11} in the first equation and z_{21} in the second. Similarly, we can constrain

$I_1 = 0$ and measure z_{12} and z_{22} . Other parameter sets require short-circuit loads at the ports. Many active devices will oscillate with such a load, and hence can't be measured under those conditions.

However, a true wideband short or open circuit directly at a port is very difficult to obtain at microwave frequencies, while a matched load can be constructed over a very broad frequency range. Although the matched termination does not constrain either V or I , it does create a situation for which there is no reflected wave regardless of the length of the transmission lines used.

This insight gave rise to the use of scattering matrix parameters, or s-parameters, to define measurements of microwave devices. Four wave (rather than V or I) variables are defined such that the square of their magnitudes represents the power in a particular wave, as

$$a_1 = \frac{V_{1i}}{\sqrt{Z_0}} \qquad a_2 = \frac{V_{2i}}{\sqrt{Z_0}}$$

$$b_1 = \frac{V_{1r}}{\sqrt{Z_0}} \qquad b_2 = \frac{V_{2r}}{\sqrt{Z_0}}$$

The two-port equations are

$$b_1 = s_{11} a_1 + s_{12} a_2$$

$$b_2 = s_{21} a_1 + s_{22} a_2$$

Note that by terminating the output port in a matched load we can constrain a_2 , the wave reflected back into the output port, such that $a_2 = 0$. We can drive the two-port from its input side and measure s_{11} and s_{21} . If we reverse the connections, we can use the same method to measure s_{22} and s_{12} .

As you read about the conversion formulas among the various parameter sets, you may get the idea of immense complexity. However, you will rarely use parameter sets other than s-parameters, since modern test equipment and device specifications are typically restricted to that format.

The study of S-Parameter matrices employs all the force of mathematical studies of matrix relationships, plus the practical interaction with modern network analyzer measurement techniques.

The current version of the HP application note on S-Parameter techniques can be downloaded in *.pdf format from the class web page.