

## S (scattering) Parameters

Scattering, or S, parameters are another extremely useful design aid that most manufacturers provide for their higher frequency transistors. While Y-parameters utilize input and output voltages and currents to characterize the operation of a two-port network, S parameters use normalized incident and reflected traveling waves in each network port. There is no need to present a short circuit to the two-port device. This termination often causes an active device, such as a transistor, to become unstable, thus making measurements impossible.

Instead, the network is always terminated in the characteristic impedance of the measuring system and this is, in most cases, purely resistive 50 Ohms. In addition, the 50-Ohm source and load seen by the two-port network forces the device under test, if active, to be stable and not oscillate.

### I. Transmission line theory.

To understand the concept of S-parameters, it is necessary to first have a working knowledge of transmission line theory.

Voltage, current, or power emanating from a source impedance  $Z_s$  and delivered to a load  $Z_L$  can be considered to be in the form of incident and reflected waves traveling in opposite directions along a transmission line of characteristic impedance  $Z_0$ .

If  $Z_L = Z_0$  then the incident wave is totally absorbed in the load.

If  $Z_L$  is not equal to  $Z_0$  then some of the incident wave is not absorbed in the load but is reflected back toward the source.

If  $Z_s = Z_0$  then the reflected wave would be absorbed by the source entirely. For  $Z_s$  not equal to  $Z_0$  the reflected from the load wave is re-reflected from the source back toward the load and the entire process repeats itself. The degree of mismatch between  $Z_0$  and  $Z_L$ , or  $Z_s$ , determines the amount of incident wave that is reflected. The ratio of the reflected wave to the incident wave is known as the **reflection coefficient**. The **reflection coefficient**,  $\Gamma$  is a complex number often expressed as a magnitude and an angle in a polar form.

$$\Gamma = V_{\text{reflected}} / V_{\text{incident}}$$

$\Gamma = 0$  if a line is terminated with a resistance equal to  $Z_0$ . A *perfect match* exists and therefore, there is no reflected wave.

$\Gamma = 1$  if the load impedance is an open, short circuit or purely reactive. All of the incident wave will be reflected back toward the source and therefore, a *perfect mismatch* exists.

So, the normal range of  $\Gamma$  is **between 0 and 1**.

For  $\Gamma > 1$  the magnitude of the reflected wave from a load must be greater than the magnitude of the incident wave to that load. The load must be a source of power in this case. This concept is useful in the design of oscillators. However, it is highly undesired in the design of input networks of amplifiers.

**Reflection coefficients** can be plotted directly on the Smith Chart and the corresponding load impedance read off of the chart without a need of any calculations. As well, given a specific characteristic impedance and a load impedance, a reflection coefficient can be read directly from the chart.

## II. S-parameters and the two-port network

The following is true for any travelling wave that originates at the source:

1. A portion of the wave from a source and incident wave upon the two port device (a1) will be reflected (b1) and another portion will be transmitted through the two-port device.

2. A fraction of the transmitted signal is then reflected from the load and becomes incident upon the output of the two-port device (a2).
3. A portion of a signal (a2) is then reflected from the output port back toward the load (b2), while a fraction is transmitted through the two-port device back to the source.

Concluding the above, any travelling wave is made up of two components. For example, the output of the two-port device to the load consists of the portion that is reflected from the output of the two-port device (a2) and the portion that is transmitted through the two-port device (a1). Similarly, the total wave flowing from the input of the two-port device toward the source consists of the portion that is reflected from the input port (a1) and the fraction of (a2) that is transmitted through the two-port device.

These observations can be illustrated in the equation form as:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad \text{and} \quad b_2 = S_{21}a_1 + S_{22}a_2$$

where,

$S_{11}$  - the input reflection coefficient  
 $S_{12}$  - the reverse transmission coefficient (reverse gain or loss)  
 $S_{21}$  - the forward transmission coefficient (forward gain)  
 $S_{22}$  - the output reflection coefficient

$$S_{11} = b_1/a_1 \quad \text{for } a_2 = 0 \text{ or } Z_L = Z_0.$$

This is an input reflection coefficient.  $S_{11}$  is equal to the ratio of a reflected wave and an incident wave with  $Z_L = Z_0$ . Thus,  $S_{11}$  can be plotted on a Smith chart and the input impedance of the two-port device can be found immediately.

Similarly,

$$S_{22} = b_2/a_2 \quad \text{for } a_1 = 0 \text{ or } Z_S = Z_0.$$

This is an output reflection coefficient that can be plotted on a Smith chart and the output impedance of the two-port device can be found immediately.

The other two S parameters are found as follows:

$$S_{21} = b_2/a_1 \quad \text{for } a_2 = 0 \text{ or } Z_L = Z_0$$

$$S_{12} = b_1/a_2 \quad \text{for } a_1 = 0 \text{ or } Z_S = Z_0$$

Notice that in order to measure the individual S parameters,  $a_1$  and  $a_2$  must be set to zero. This is easily done by terminating a network (source and load) or forcing  $Z_S$  and  $Z_L$  to be equal to the characteristic impedance of the measuring system, thus eliminating all reflections from the termination.

**S-parameters** are expressed in **Re/Im, Mag/Phase or dBMag/Phase**. Note that dBMag is calculated as  $20\log_{10}\sqrt{R^2 + X^2}$  for all s-parameters.

### III. SMALL SIGNAL AMPLIFIER DESIGN:

Transistors can be completely characterized by their S parameters that vary with frequency and bias level. With these parameters, it is possible to calculate potential instabilities (tendency to oscillation), maximum available gain, input and output impedances, and transducer gain. It is also possible to calculate optimum source and load impedances either for simultaneous conjugate matching or simply to help choose specific source and load impedances for a specified transducer gain.

#### STABILITY:

To calculate the stability of a transistor with S parameters, the **intermediate quantity, Ds** must be obtained first:

$$(\text{Delta}) D_s = S_{11}S_{22} - S_{12}S_{21}$$

Then, Rollet Stability Factor, K is derived as follows:

$$K = (1 + |D_s|^2 - |S_{11}|^2 - |S_{22}|^2) / (2|S_{21}||S_{12}|)$$

If  $K > 1$ , then the device will be *unconditionally stable* for any combination of source and load impedances. For  $K < 1$  the device is *potentially unstable* and will most likely oscillate with certain combinations of source and load impedance. If that's the case, choose source and load impedances very carefully or select another bias point for the transistor or choose a different transistor. Note that the stability condition is frequency dependant.

### ***Stability circles:***

For  $K < 1$  (**potentially unstable** network) it is very important to choose source and load impedances very carefully. One of the best methods of determining those source and load impedances that will cause transistor to become unstable is to plot *stability circles* on a Smith Chart. A stability circle represents the boundary between those values of source and load impedance that cause instability and those that do not. The perimeter of the circle thus represents the locus of points which forces  $K = 1$ . Either the inside or the outside of the circle may represent unstable region.

#### ***Input stability circle (source):***

1. Calculate  $D_s$  first.
2. Calculate  $C_1$  as  $C_1 = S_{11} - D_s S_{22}^*$
3. Calculate the Center location of the circle:  $C_s = C_1 / (|S_{11}|^2 - |D_s|^2)$
4. Calculate the Radius of the circuit:  $r_s = |(S_{12} S_{21}) / (|S_{11}|^2 - |D_s|^2)|$

#### ***Output stability circle (load):***

1. Calculate  $C_2$  as  $C_2 = S_{22} - (D_s S_{11})^*$
2. Calculate the center location of the circle:  $C_l = C_2^* / (|S_{22}|^2 - |D_s|^2)$
3. Calculate the radius of the circle:  $r_l = |(S_{21} S_{12}) / (|S_{22}|^2 - |D_s|^2)|$

\* The asterick indicates the conjugate of eg.  $C_2$ ,  $S_{11}$  (same magnitude but opposite sign for the angle).

Consider the origin of an X – Y plane to be at the center of the Smith Chart.

Note that  $C_s$  and  $C_l$  are usually presented in the polar form,  $C_s = |C_s| \angle \theta$  and  $C_l = |C_l| \angle \theta$  where  $|C_s|$  and  $|C_l|$  are the distances from the center of the Smith Chart.

Plot the stability circles on the chart.

Note that  $S_{11}$  and  $S_{22}$  are less than 1 for a *stable transistor* and greater than 1 for an *unstable transistor*. It is very rare, however, to find a transistor that is unstable with a 50 Ohm source and load.

**If  $S_{11}$  and  $S_{22} < 1$**  (where 1 is the magnitude) then the center of the normalized Smith Chart represents a *stable point*. Therefore, if one of the circles surrounds the center of the chart, the inside of that circle represents the region of *stable* impedances for that port. If the circle does not surround the center of the Chart, then the entire area outside of that circle but inside the Smith Chart represents the *stable* operating region of that port.

**If  $S_{11}$  and  $S_{22} > 1$**  then the center of the Smith Chart represents an unstable point.

For an **unconditionally stable** network, not only  $S_{11}$  and  $S_{22}$  are less than 1 but also the entire Smith Chart represents a stable operating region as the stability circles fall outside the Chart.

## Gain Circles

### Unilateral Input/Output Gain Circle

Unilateral Figure of Merit,  $U$ .

All two-port models are bilateral, so both the forward and reverse signal flow must be considered. If the signal flow in the reverse direction is much smaller than the flow in the forward direction then it's possible to make the simplification that the reverse flow is zero. The Unilateral Figure of Merit,  $U$  is a quick calculation that can be used to determine where this simplification can be made without significantly affecting the accuracy of the model.

A transistor is said to be unilateral if the signal/power bounced back from the output to the source is equal to 0. This occurs if the reverse transmission coefficient,  $S_{12}$  or the reverse transducer power gain,  $|S_{12}|^2$  is equal to 0.

In order to determine how close to ideal unilateral a network is, the Unilateral Figure of Merit,  $U$  has been defined as follows:

$$U = (|S_{12}||S_{21}||S_{11}||S_{22}|)/[(1-|S_{11}|^2)(1-|S_{22}|^2)]$$

Error limits on Unilateral calculations:

$$1/(1+U)^2 < G_T/G_{TU} < 1/(1-U)^2$$

where,

**$G_T$  – transducer power gain**, defined as the ratio of the output power delivered to a load by a source and the maximum power available from the source. It includes the effects of input and output impedance matching. Component resistive losses are neglected.

$$G_T = |S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)/[(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{12}S_{21}\Gamma_L\Gamma_S]^2$$

**$G_{TU}$  – transducer power gain** assuming unilateral property,  $S_{12} = 0$ .

The unilateral gain circles are developed to identify the best load and source impedances to minimize the error due to unilateral assumption.

#### Input Gain Circle:

1. Calculate gain  $G_s$ :  $G_s = (1-|\Gamma_S|^2)/(|1-S_{11}\Gamma_S|^2)$

Note: Multisim RF simulator allows to enter  $G_s$  manually.  $G_s$  is expressed in dB.

2. Calculate  $g_s$ :  $g_s = G_s(1-|S_{11}|^2)$ .

The magnitude of  $G_s$  is used in the above mentioned equation.

3. Calculate the center location of the circle:  $C_s = g_s S_{11}^*/(1-(1-g_s)|S_{11}|^2)$

4. Calculate the radius of the circle:  $R_s = \sqrt{1-g_s(1-|S_{11}|^2)/(1-(1-g_s)|S_{11}|^2)}$

Note: All points on the circle represent the same gain for different input impedances.

#### Output Gain Circle:

1. Calculate gain  $G_L$ :  $G_L = (1-|\Gamma_L|^2)/(|1-S_{22}\Gamma_L|^2)$

Note: Multisim RF simulator allows to enter  $G_L$  manually. It is expressed in dB.

2. Calculate  $g_l$ :  $g_l = G_L(1-|S_{22}|^2)$ .

The magnitude of  $G_L$  is used in the above mentioned equation.

3. Calculate the center location of the circle:  $C_L = g|S_{22}|^2 / (1 - |g| |S_{22}|^2)$

4. Calculate the radius of the circle:  $R_L = \sqrt{1 - |g| (1 - |S_{22}|^2) / (1 - |g| |S_{22}|^2)}$

Note: All points on the circle represent the same gain for different output impedances.

For maximum gain the circles turn into a dot on Smith Chart.  $G_{\text{max}} = 1 / (1 - |S_{11}|^2)$  and  $G_{\text{Lmax}} = 1 / (1 - |S_{22}|^2)$ . The circles would pass through the center of the Smith Chart if  $G_s = G_L = 1$  or 0 dB gain. The input and output impedances are equal to 50 Ohms. So, the complete matching has been achieved.

Matching impedances would provide maximum power transfer from the input to the output for a very narrow frequency range. Sometimes, designers intentionally introduce mismatching at either input or output to maintain the same power gain (which is less than the maximum power gain) for a wider frequency range losing some power.

#### The steps to design amplifier using Input and Output Gain Circles:

1. Identify the gain of the mismatched network (a transistor),  $|S_{21}|^2$ . This gain is a constant value.

2. Calculate  $G_{\text{max}}$  and  $G_{\text{Lmax}}$

3. Select  $G_s$  and  $G_L$  so that  $G_s < G_{\text{max}}$  and  $G_L < G_{\text{Lmax}}$  and the desired gain is achieved.

Note that the desired gain in dB is:  $G_{\text{overall}} = G_s + G_L + 10 \log_{10} |S_{21}|^2$

4. Select the closest point on the circles to the center of the Smith Chart to minimize the mismatch and maximize the bandwidth. These points will identify the input and output matching impedance for the desired power gain.

#### POWER GAINS:

Multisim RF Simulator calculates Power Gain (PG), Available Power Gain (APG) and Transducer Power Gain (TPG) for  $Z_o = 50 \Omega$  at a given frequency. The dBMag is derived as  $10 \log_{10} |PG|$ .

PG is defined as the ratio of the power delivered to the load and the average power delivered to the network from the input.

RF simulator assumes  $Z_L = Z_o$  and  $Z_s = Z_o$  or  $\Gamma_s = \Gamma_L = 0$ .

Therefore,

$$PG = |S_{21}|^2 / (1 - |S_{11}|^2).$$

The Transducer Power Gain, TPG is the ratio of the power delivered to the load to the power available from the source.

$$\text{For } \Gamma_s = \Gamma_L = 0, \quad TPG = |S_{21}|^2.$$

The Available Power Gain, APG is the ratio of the power available from the output port of the network to the power available from the source and it is expressed as

$$APG = |S_{21}|^2 / (1 - |S_{22}|^2)$$

#### VOLTAGE GAIN:

Voltage Gain, VG is obtained for  $\Gamma_s = \Gamma_L = 0$  and is expressed as

$$\mathbf{VG} = \mathbf{S}_{21}/(\mathbf{1} + \mathbf{S}_{11})$$

Voltage Gain expressed in dBMag is calculated as  $20\log_{10} |\mathbf{VG}|$ .