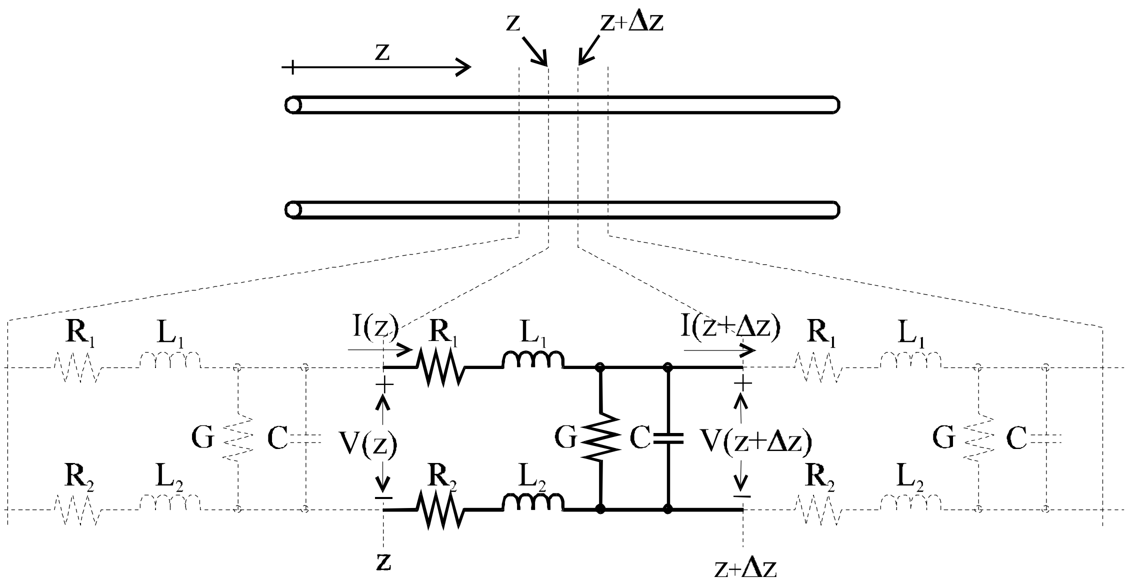
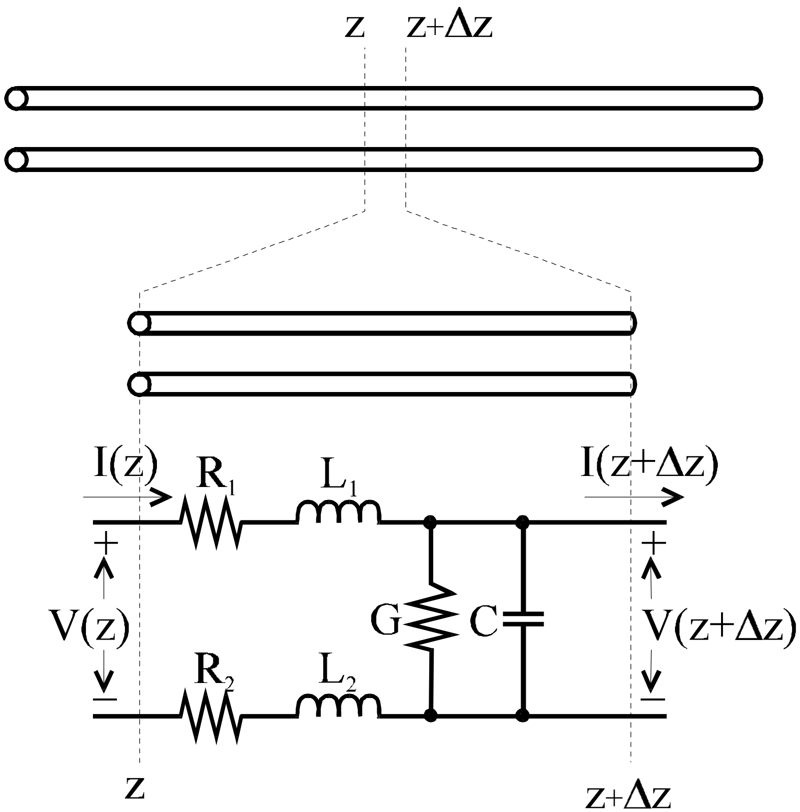
General Transmission Line Equations

- Detailed analysis of a differential section



Note: Analysis applies to all types of transmission lines such as coax cable, two-wire, microstrip, etc.

Kirchhoff's laws on a microscopic level



- Over a differential section we can again use basic circuit theory
- Model takes into account line losses and dielectric losses
- Ideal line involves only L and C

Advantages versus disadvantages of electric circuit representation

- Clear intuitive physical picture
- yields a standardized two-port network representation
- serves as building bocks to go from microscopic to macroscopic forms

- Basically a one-dimensional representation (cannot take into account interferences)
- Material nonlinearities, hysteresis, and temperature effects are not accounted for

Derivation of differential transmission line form

KVL:

$$V(z) = (R + jwL)I(z)\Delta z + V(z+\Delta z)$$

$$-Lim\left(\frac{V(z+\Delta z) - V(z)}{\Delta z}\right) = \boxed{-\frac{dV(z)}{dz} = (R + jwL)I(z)}$$

KCL:

$$I(z) = (G + jwC)\Delta z V(z+\Delta z) + I(z+\Delta z)$$

$$\boxed{-\frac{dI(z)}{dz} = (G + jwC)V(z)}$$

Coupled DE

Traveling Voltage and Current Waves

$$\frac{d^2V(z)}{dz^2} - k^2V(z) = 0 \quad \longleftrightarrow \quad \frac{d^2I(z)}{dz^2} - k^2I(z) = 0$$

where

$$k = k_r + jk_i = \sqrt{(R + jwL)(G + jwC)}$$

Phasor expressions

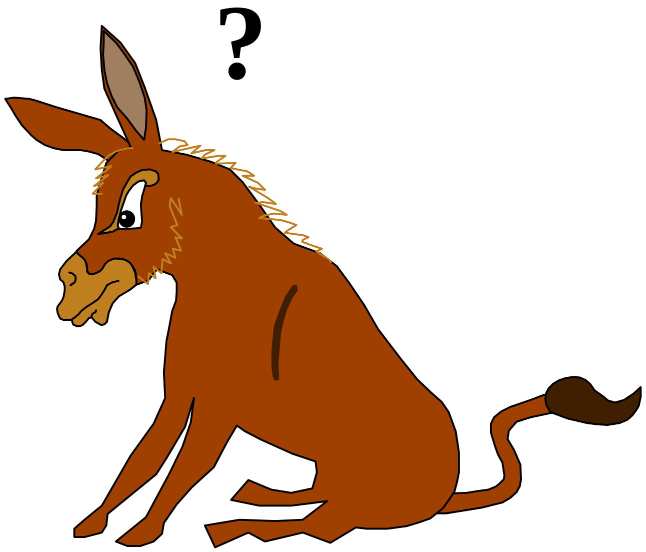
Right traveling wave

$$V(z) = \boxed{V^+e^{-kz}} + \boxed{V^-e^{+kz}} \qquad I(z) = \boxed{I^+e^{-kz}} + \boxed{I^-e^{+kz}}$$

Left traveling wave

General line impedance definition

$$I(z)=\frac{k}{(R+jwL)}(V^{+}e^{-kz}-V^{-}e^{+kz})$$



$$Z_0=\sqrt{\frac{(R+jwL)}{(G+jwC)}}=\frac{V^{+}}{I^{+}}=-\frac{V^{-}}{I^{-}}$$

Characteristic line impedance